Table 1 Plastic notch parameters for a titanium alloy structural element with $K_{el} = 2$

	Eq. (4)			Eq. (9)			Discrepancy, %	
p/σ_y	σ/σ_y	K_p	K_ϵ	σ/σ_y	K_p	K_ϵ	K_p	K_ϵ
0.598	1.00	1.67	2.39	1.00	1.67	2.39	0	0
.802	1.06	1.33	3.02	1.10	1.37	2.93	2.9	3.1
.889	1.08	1.22	3.28	1.14	1.28	3.13	4.7	4.8
1.000	1.12	1.12	3.56	1.24	1.24	3.24	9.7	9.9
1.100	1.19	1.08	3.69	1.36	1.24	3.24	12.9	13.9

of the various notch parameters as p/σ_y is incremented, simulating a monotonic loading test of a notched tensile specimen.

There is a limitation, however, on the applicability of Eq. (4) based upon the limitations of the Ramberg-Osgood equation in describing stress-strain properties of materials. Analysis of many sets of stress-strain data for steel, titanium, and aluminum have revealed that increasingly larger discrepancies between test data and calculated values occur for data points beyond the yield stress. Post-yield data require a different analytic formulation.[2] Hence, Eq. (4) should not be used for values of (σ/σ_y) greater than unity. If this criterion is applied to the results shown in Figure 2 of Ref. 1, then the following values of (p/σ_y) represent the upper limit of applicability of the analysis:

M value =	3	5	9	199
(p/σ_y) limit =	0.561	0.583	0.596	0.598

The semigraphical K_p results given in the figure for p/σ_y values in excess of those given above must be considered questionable.

It has previously been shown[2] that stress-strain data beyond the yield can be approximated by

$$A = (0.7E/\sigma_y)^n \sigma_y \tag{7}$$

After substitution of Eq. (7) into Eq. (6) and algebraic manipulation of the resulting equation, an analog to Eq. (2) is derived in the form

$$\sigma\epsilon = (\sigma_y^2/E)(10/7)(\sigma/\sigma_y)^a \tag{8}$$

where $a = (1+n)/n$.

For values of p/σ_y less than unity, the equation for σ/σ_y is obtained by substituting Eqs. (8) and (3) in Eq. (1) which after rearrangement becomes

$$\sigma/\sigma_y = (.7K_{el}^2[(p/\sigma_y)^2 + (3/7)(p/\sigma_y)^{m+1}])^{1/a} \tag{9}$$

For values of p/σ_y equal to or greater than unity, the values of σ/σ_y is given by

$$\sigma/\sigma_y = K_{el}^{2/a}(p/\sigma_y) \tag{10}$$

Note that Eq. (10) indicates that K_p and K_ϵ reach limiting values when the reference stress reaches the secant yield stress. The limiting values are given by

$$K_p = K_{el}^{2n/(1+n)} \tag{11a}$$

$$K_\epsilon = K_{el}^{1/(1+n)} \tag{11b}$$

An analysis of tension stress-strain data from titanium alloy TI-8-8-8-2-3 was made using the methods of Ref. 2. The exponents found were $m = 19.16$ and $n = 0.18090$.

The stress and strain parameters for $K_{el} = 2$ were calculated for incremental p/σ_y values using Eq. (4) and Eq. (9) with the initial p/σ_y chosen to yield $\sigma/\sigma_y = 1$. The comparison is shown in Table 1. The values calculated from Eq. (4) are the same as those which would be obtained from the semigraphical method of Ref. 1.

The discrepancies between the results is a quantitative indication of the limitation of the Ramberg-Osgood equation in describing the stress-strain properties beyond the yield of the particular titanium alloy chosen. Other materials, particularly those with low m values exhibit the same characteristics.

Two final comments. The authors caution that the semigraphical method may be increasingly more inaccurate for large K_{el}. The direct algebraic method using Eq. (4) or Eq. (9) has no such limitation.

In response to the conclusion of Ref. 1 regarding experimental verification of the Neuber equation, I would refer the authors to an experimental study of notch strain.[3] Comparison between theoretical results, developed using the Neuber equation with a piecewise analytical approximation of stress-strain properties, and experiments indicated that for K_{el} of 1.5 and 2.0, the theory was conservative. Subsequent unpublished results for larger K_{el} up to 6.0 showed the same trend. Additionally, the experiments showed that for any K_{el} value the discrepancy between theory and experiment increased as the load level was increased. The discrepancy may, in part, be the result of using uniaxial stress-strain properties in developing the theoretical results for a stress field in the test specimens which is essentially biaxial.

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Comment on "A Mathematical Description of Gas-Surface Interactions Based on Reciprocity"

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KINSLOW[1] uses the reciprocity principle to develop an interesting model for a scattering kernel to describe reflected velocity distributions in gas-surface interactions.

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Although he presents results that are in excellent agreement with measurements by Moran[2] in the thermal scattering regime, and he offers a model that is capable of a general statement of the rarefied boundary conditions, it appears that Kinslow's model is based on an assumption that cannot be valid for a large number of cases, if ever. Both the regime and the type of measurements Moran made tend to obscure the consequences of the challenged assumption.

Kinslow states that the reciprocity condition on the scattering kernel, although derived from the requirements of equilibrium between gas and surface, must be generally valid for individual molecular encounters, and therefore for nonequilibrium scattering as well. This statement does not appear to be unreasonable as a basis for a model if one excludes the case of adsorption (also known as sticking or trapping) or desorption of previously adsorbed particles. He then proceeds to the assumption that the scattering kernel, $P(v',v)$, can be written as the product of all of the component kernels, $P_i(v_i',\ v_i)$. He bases this assumption on an extension of Maxwell's "idea that each velocity component in a gas is independent," from which he proceeds to the assumption that "the interaction of each velocity component with the surface is independent." It is apparent that this cannot be the case, at least in the limit of structural scattering of an incident molecular beam, which is the case of interest in space vehicle aerodynamics and in some of the high energy impacts of current interest for fusion power reactors and their equipment.

The reader is referred to Refs. 3-8, in which the principles of the thermal and structural scattering regimes were discovered[3] and developed[4] by extensive numerical experiments, and verified by laboratory experiments of at least four teams of investigators.[5-8] In the structural scattering regime the surface atoms comprise an array of particles, the aggregate potential surface of which appears much more irregular to an incident particle of high kinetic energy (compared to the characteristic pairwise binding energy) than to one of low energy. In thermal scattering the thermal motion of the solid atoms dominates over this structure so that collisions are much more like gas-gas encounters. It can be shown fairly easily[9] that in the structural domain thermal motion is insignificant, recoil in the lattice can be dealt with by considering each impact atom as supported by a separate foundation, and each impact atom acts as a scattering center. This implies a nearly conservative force field, with energy extraction from the incident particle possible only by recoil of the impacted atoms. In such a scenario the components of the velocity vector of a scattered particle are each directly influenced by the complete incident velocity vector. Kinslow's model would appear to require that there be no significant memory of the incident energy, thereby requiring both geometric surface roughness and microscopic roughness in the potential surface due to the presence of discrete atoms to be unimportant compared to lattice thermal scattering.

As a case in point, let us consider out-of-plane scattering. In Kinslow's model there can be no variation in the relative amount of out-of-plane scattering with energy or incident angle, yet such variations have been predicted,[4] measured (Calia and Oman[7]) and shown to be significantly different in different situations.[4,7] The proper modeling of out-of-plane scattering is essential to determination of normal and tangential momentum interactions. Moran's thermal scattering data, like most gas scattering measurements, gives only arbitrarily scaled relative intensity, and only in the plane of incidence. Without knowledge of the absolute fraction of the incident beam scattered into each solid angle, no model for the scattering kernel can be verified (or disproved). The data of Calia and Oman[7] include reflected velocity distributions and absolute determinations of the local scattered to incident density fields, both in and out of the plane of incidence.

It may be possible to replace Kinslow's Eq. (5) by an equation of the form

$$P(v',v) = P(v',v_x',v_x)\,P(v',v_y',v_y)\,P(v',v_z',v_z) \qquad (1)$$

which would preserve the ability to model structural scattering. In addition to the data and numerical experiments already mentioned, numerical experiments by several teams are reviewed by Goodman,[10] and these should also be considered as sources for verifying scattering kernel models. I believe such an extension would make Kinslow's model a much a more valuable tool.

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Reply by Author to R.A. Oman

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OMAN, in effect, challenges the assumption made in Ref. 1 that the scattering kernel can be represented as a product of independent probability functions, one for each coordinate direction. He presents no basis for this challenge except the statement, "It is apparent that this cannot be the case, at least in the limit of structural scattering..." He then attempts to discredit my model with the statement, "In Kinslow's model there can be no variation in the relative amount of out-of-plane scattering with energy or incident angle..." It is believed the statements made by Oman concerning the product kernel were based on an incomplete understanding of the scattering kernel as given by Eq. (16) in

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